

Dynamics of Liquid Agitation in the Absence of an Air-liquid Interface

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The dynamics of agitating single- and two-phase liquid mixtures in the absence of an air-liquid interface was compared with that of similar systems with an interface. Two geometrically similar cylindrical vessels, 12 and 18 in. in diameter respectively, were used, each fitted with a similar six-bladed, disk-turbine impeller.

New general correlation curves of power number with Reynolds number are presented which show that elimination of the air-liquid interface makes it possible to attain dynamic similarity in scale-up in unbaffled vessels but makes little difference in baffled vessels.

In the study of the dynamics of liquid agitation, particular attention has been given to open baffled vessels operated batchwise with a single liquid phase (2, 6). Where study or design of such a system as a continuously operated, multiple-stage, liquid-liquid extractor must be considered, there is little information available on the effect of operation with continuous flow of two liquid phases in the absence of an air-liquid interface above the impeller.

Olney (4) recently reported that in mass transfer studies in such an extractor, higher stage efficiencies were obtained with no baffles than with baffles at the same power input. However, earlier agitation studies based on open unbaffled vessels indicate that with turbulent agitation it is not possible to scale up an unbaffled extractor and obtain dynamic similarity in a larger system (3, 5). The flow dynamics giving optimum results in a pilot plant therefore could not be reproduced in plant-scale equipment which is geometrically similar.

In an open unbaffled vessel, turbulent agitation will result in a vortex. The flow characteristics are affected by the geometry of the vessel and the impeller, the impeller speed, and the properties of the liquid. These can be related for geometrically similar systems by

$$\frac{P g_c}{D^5 N^3 \rho} = K \left(\frac{D^2 N \rho}{\mu} \right)^m \left(\frac{D N^2}{g} \right)^n \quad (1)$$

or

$$N_p = K(N_{Re})^m (N_F)^n \quad (2)$$

where the values of K , m , and n are characteristics of the type of impeller (6). Here a dimensionless power number, N_p , is expressed as a function of the dimensionless agitation Reynolds number (for viscous forces in the liquid) and the Froude number (for gravitational forces producing the vortex). It can be shown mathematically that dynamic similarity cannot be obtained in scale-up of a

system with both these forces controlling the flow pattern (5).

The vortex is eliminated by adding radial baffles to the vessel. With the gravitational forces negligible, the flow dynamics are controlled by the viscous forces in the liquid, and Equation (2) becomes (6)

$$N_p = K(N_{Re})^m \quad (3)$$

Thus for baffled vessels dynamic similarity is obtained in two geometrically similar vessels operated at the same Reynolds number.

In a covered unbaffled vessel with no air-liquid interface the swirl flow pattern characteristic of unbaffled operation is obtained; however, there is no vortex. The equations derived for open vessels do not describe this type of agitation.

In the determination of the Reynolds and power numbers for agitation of two-phase liquids, mean values of density and viscosity are used. Miller and Mann (3) recommended the use of a weighted geometric mean viscosity, derived from studies in open unbaffled vessels:

$$\mu_a = \mu_x^x \mu_y^y \quad (4)$$

TABLE 1. PHYSICAL PROPERTIES OF LIQUIDS STUDIED

Liquid	Temperature, °F.	Density, g./ml.	Viscosity, centipoises
Water	68	0.9982	1.000
S.A.E. 10 motor oil	68	0.8914	96.35
Kerosene I	68	0.8132	2.271
Kerosene II	68	0.8144	2.121
Blend A (kerosene and oil)	68	0.8600	21.22
Water saturated with sec-butanol	77	0.9720	1.931
Sec-butanol saturated with water	77	0.8742	4.367
Glycerine (aq.)	86	1.134	4.62
S.A.E. 30 motor oil	77	0.9292	368.5
Blend D (kerosene and oil)	86	0.9032	35.94

Vermeulen and associates (7) have recommended another relationship, derived from studies in baffled vessels with no air-liquid interface:

$$\mu_m = \frac{\mu_c}{1 - x_d} \left[1 + \frac{(1.5)(x_d)(\mu_d)}{\mu_c + \mu_d} \right] \quad (5)$$

Vermeulen also recommends for density the use of a weighted arithmetic mean, originally used by Miller and Mann:

$$\rho_a = x \rho_x + y \rho_y \quad (6)$$

These empirical equations may be satisfactory for the physical systems for which they were derived, but there is no theoretical basis to evaluate their reliability for open baffled vessels or covered unbaffled vessels with no air-liquid interface.

No information has been published on the quantitative effect of continuous flow on the power characteristics of impellers.

It is apparent that a need exists for the basic correlations necessary to define the dynamics of agitation in the absence of an air-liquid interface. Flynn (1) started the investigation with a study of mass transfer rates in covered baffled extractors. This investigation continues that work with the emphasis on liquid dynamics.

APPARATUS AND SCOPE OF EXPERIMENTS

Correlations are based on power-speed measurements taken with batch and continuous-flow operation with both single- and two-phase liquids. All runs in unbaffled vessels were made in the absence of an air-liquid interface. For baffled vessels runs were made with and without an interface for comparative purposes. The liquids and mixtures studied are listed in Tables 1, 2, and 3 along with the range of operating conditions (1a).

The vessel and impeller are shown in Figure 1. Two geometrically similar cylindrical vessels 12 and 18 in. in diameter respectively were used. Each was fitted with a similar six-bladed, disk-turbine impeller, manufactured by the Mixing Equipment Company of Rochester, New York. This type of impeller was selected because of its wide application in liquid-liquid extractors. All data were taken with the impeller at the midpoint of the vessel, except where the effect of impeller height was under study.

Baffles, when used, were 16.7% of the vessel diameter. This choice of baffle width was dictated by the results obtained by Flynn (1), who measured the effect on power of operation with and without an

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TABLE 2. SUMMARY OF SYSTEMS AGITATED UNDER BATCH CONDITIONS

Liquid system	Volumetric phase ratio	Air-liquid interface	Impeller C/D	Impeller speed, rev./sec.
<i>Unbaffled 12-in. vessel with 4-in. impeller</i>				
Water	—	No	1.5	6.30-25.20
S.A.E. 10 motor oil	—	No	1.5	5.86-22.00
Kerosene I	—	No	1.5	11.11-24.84
Blend A	—	No	1.5	7.89-22.52
Water-S.A.E. 10 motor oil	0.5-6	No	1.0-2.0	5.81-24.60
Water-kerosene-I	0.178-10	No	1.5-2.0	8.33-24.94
Water-Blend-A	1.04	No	1.5	10.06-22.94
<i>Unbaffled 18-in. vessel with 6-in. impeller</i>				
Water	—	No	1.5	4.46-9.50
Kerosene II	—	No	1.5	5.08-10.50
Glycerine (aq.)	—	No	1.5	3.50-9.42
S.A.E. 30 motor oil	—	No	1.5	2.85-7.60
Water-kerosene-II	0.35-2.65	No	1.5	4.16-11.60
<i>Baffled 12-in. vessel with 4-in. impeller</i>				
Water	—	Yes	1.5	4.09-8.37
	—	No	1.5	5.02-8.36
S.A.E. 10 motor oil	—	Yes	1.5	5.12-19.62
	—	No	1.5	5.91-19.05
Kerosene I	—	No	1.5	6.24-18.11
Blend A	—	No	1.5	6.07-17.44
	—	Yes	1.5	6.12-15.10
Water-S.A.E.-10 motor oil	0.5-2.0	No	1.5-2.0	4.42-16.37
	0.5-1.13	Yes	1.5	4.42-15.72
Water-kerosene-I	0.49-4.25	No	1.5	5.04-16.41
Water-Blend-A	1.06	No	1.5	5.25-15.57
	1.20	Yes	1.5	6.38-11.62
<i>Baffled 18-in. vessel with 6-in. impeller</i>				
Blend D	—	No	1.5	2.43-6.05
Water-Blend-D	0.56-0.85	No	1.5	3.75-5.72

TABLE 3. SUMMARY OF SYSTEMS AGITATED UNDER CONTINUOUS-FLOW CONDITIONS (No Air-liquid Interface, C/D = 1.5)

Liquid system	Volumetric phase ratio	Impeller speed, rev./sec.	Flow rate, lb./min.
<i>Unbaffled 12-in. vessel with 4-in. impeller</i>			
Water	—	10.90-20.72	76
	—	11.52-23.50	28
Water-ke-kerosene-II	1.22-1.44	10.82-22.90	6.4
<i>Unbaffled 18-in. vessel with 6-in. impeller</i>			
Water	—	4.26-7.79	77
<i>Baffled 12-in. vessel with 4-in. impeller</i>			
Water	—	5.13-13.03	79
<i>Baffled 18-in. vessel with 6-in. impeller</i>			
Water-blend-D	0.51-0.79	3.74-5.76	12

air-liquid interface and with 10, 16.7, and 25% baffles. He found that 16.7% baffles were the largest for which differences in power requirements were obtained for the two methods of operation. It was one of the objectives of this study to confirm and refine Flynn's results by use of larger vessels, the power requirements of which could be measured more accurately. When, as will be shown, no difference was found between operation with and without an air-liquid interface at 16.7% baffles, it

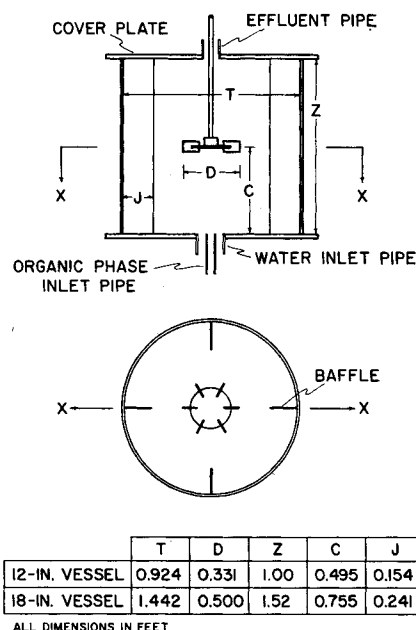


Fig. 1. Mixing vessel and disk-turbine impeller.

was not considered justified to pursue this part of the study further with the more customary 10% baffles.

Power was measured with a torque-table dynamometer, similar to that employed by previous investigators. It consisted of the motor supported by a thrust bearing and

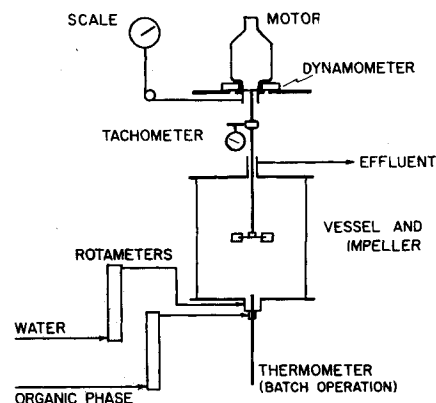


Fig. 2. Diagram of equipment.

restrained by a cord attached to a dynamometer scale and auxiliary weights (Figure 2).

Continuous flow was provided by a suitable arrangement of feed tanks, piping, a pump, rotameters, and receiving tanks. Runs were made with batch operation except where the effect of continuous flow was evaluated.

UNBAFFLED VESSELS

For turbulent agitation of a liquid in an open, unbaffled vessel, a plot of power number against the Reynolds number alone gives a family of curves with speed as the parameter (6). The Froude number must be included to provide a single, continuous curve.

All power measurements in this study of unbaffled vessels were taken with a full covered vessel without an air-liquid interface or a vortex. A review of all such measurements for any given liquid agitated at a number of different speeds showed that the correlation of power number with Reynolds number alone resulted in a smooth, continuous curve. The use of the Froude number was not required as it is for open unbaffled vessels. This observation is valid for all the data for unbaffled vessels, regardless of liquids, phase ratios, impeller height, flow conditions, or vessel size.

In addition, a general correlation of power number with Reynolds number for all batch systems resulted in a single, general curve which described all the single-phase and most of the two-phase liquids studied. This curve is shown in Figure 3 as it was derived from the data points for single-phase liquids in both the 12- and 18-in. vessels.

The significance of these observations is considerable. Until now Rushton and other investigators have emphasized the necessity of studying agitation only in baffled vessels because of the impracticability of scale-up with both gravitational and viscous forces controlling the flow pattern (5). The results of this study now show that laboratory studies of mass or heat transfer need not be limited to baffled systems. By operation without an air-liquid interface, the possible

advantages of unbaffled agitation can be evaluated. The flow dynamics giving optimum results on the small scale can be reproduced in a larger system which is geometrically similar.

Of the general equations relating the Reynolds and power numbers, Equation (2), originally derived for baffled vessels, can be used, as the effect of gravitational forces is eliminated. Between $N_{Re} = 10^3$ and $N_{Re} = 10^5$, the general correlation curve is approximately straight, and so the constant K and slope m can be evaluated to give

$$N_P = 14.86(N_{Re})^{-0.2545} \quad (7)$$

For other values of N_{Re} , as the slope is variable, single quantitative values cannot be substituted for m and K .

Two Liquid Phases

Neither the weighted geometric mean viscosity μ_a of Miller and Mann nor the mean viscosity μ_m of Vermeulen was satisfactory in the correlation of data for two-phase liquids with data for single-phase liquids for this investigation. The average deviation of the power numbers from those given by the curve for single-phase liquids in these cases were 23.2% (for μ_a) and 16.1% (for μ_m). Instead, modifications of Vermeulen's viscosity were found to fit the data best as follows:

For water more than 40% by volume:

$$\mu_L = \frac{\mu_w}{x_w} \left[1 + \frac{6.0x_o\mu_o}{\mu_w + \mu_o} \right] \quad (8)$$

For water less than 40% by volume:

$$\mu_L = \frac{\mu_o}{x_o} \left[1 - \frac{1.5x_w\mu_w}{\mu_w + \mu_o} \right] \quad (9)$$

These equations cover cases where the impeller was located both above, below, and at the liquid-liquid interface for the system at rest. In Figure 4 the two-phase liquid data points calculated with μ_a and μ_L are superimposed on the general correlation curve for single-phase liquids. The average deviation of the power numbers from those given by the curve is 7.7%.

It is noted that individual points at $N_{Re} = 10^4$ show greater deviation than those at $N_{Re} = 10^5$. These deviations are attributed to lack of uniformity of the two-phase mixture at low impeller speeds. In every case the points coincide with or approach the general curve at higher speeds.

Effect of Impeller Height

The effect of the impeller height with relation to the liquid-liquid interface was evaluated by moving the impeller in steps from the heavy to light phase of various two-phase systems. Between heights of one to two impeller diameters from the bottom, substantially no effect was found.

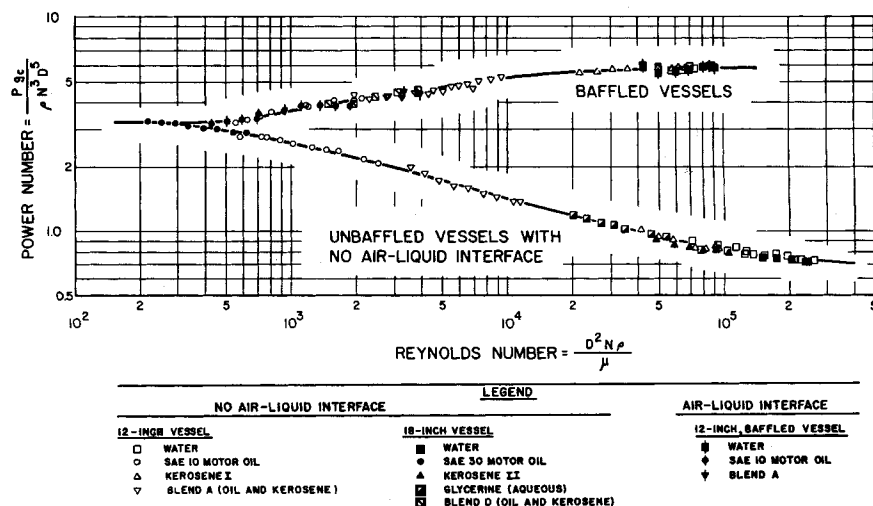


Fig. 3. General correlation of power number with Reynolds number for single-phase liquids agitated by batches in baffled and unbaffled vessels.

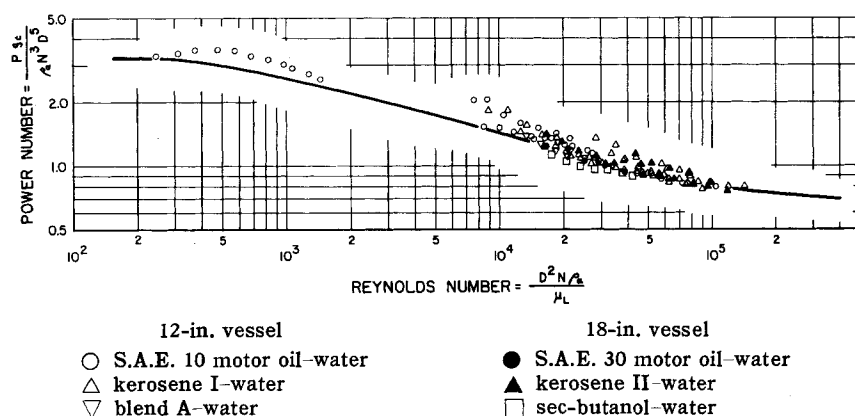


Fig. 4. Correlation of two-phase liquid data points with the general curve for single-phase liquids, agitated by batches in unbaffled vessels with no air-liquid interface.

Continuous-flow Operation

Flow through the vessel was found to have a small but measurable effect on power number. The deviations in the 12- and 18-in. vessels can be related linearly to (water flow rate)/(vessel diameter) as in Figure 5. The flow rates employed in water runs were high, corresponding to a retention time as low as 32 sec. Measurements with kerosene-water at the more normal retention time of 7 min. agreed with those for batch operation. For most applications, therefore, the effect of continuous flow on agitation dynamics can be ignored.

BAFFLED VESSELS

Essentially all data taken in baffled vessels operated batchwise were correlated by a single general curve similar to that developed by Rushton for single liquids in open vessels (6). The only exceptions were deviations caused by air entrainment in open vessels with agitation at power inputs (20 to 80 hp./1,000 gal.) far above those normally encountered in practice.

The correlation obtained from this study is shown in Figure 3 as it was

derived from the measurements with single-phase liquids. It should be noted that this curve includes runs made both with and without an air-liquid interface.

It is concluded, therefore, that there is substantially no effect on liquid dynamics between operation with and without an air-liquid interface in a vessel with 16.7% baffles.

Two Liquid Phases

Power measurements for two-phase liquids in baffled vessels were correlated with the general curve with the mean viscosity of Vermeulen [Equation (5)] and the weighted arithmetic mean density [Equation (6)]. In Figure 6 the calculated values of N_p and N_{Re} for two-phase liquids are shown superimposed on the general curve derived for single-phase liquids. The continuous phase, for the purpose of calculating μ_m , was taken as that phase in which the impeller was located when at rest.

The weighted geometric mean viscosity [Equation (4)] of Miller and Mann was in no case completely satisfactory for correlation of two-phase liquid measurements in baffled vessels. When a high-viscosity phase such as oil is continuous,

the deviations resulting from use of this function are large.

Effect of Impeller Height

In contrast to unbaffled vessels, in baffled vessels impeller height in relation to the liquid interface was found to have a significant effect.

Though the data taken on the effect of impeller height were too limited to warrant completely general conclusions, it was observed that the continuous phase

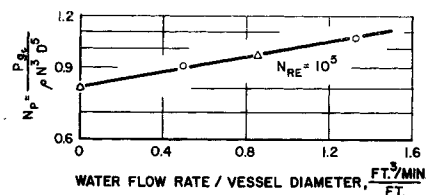


Fig. 5. Deviation of power number with water flow rate through unbaffled vessels. \circ 12-in. vessel, \triangle 18-in. vessel.

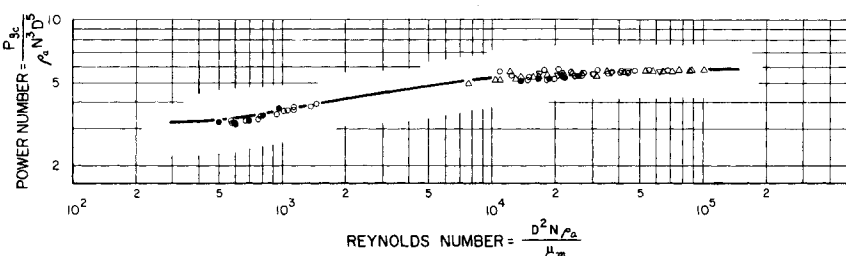
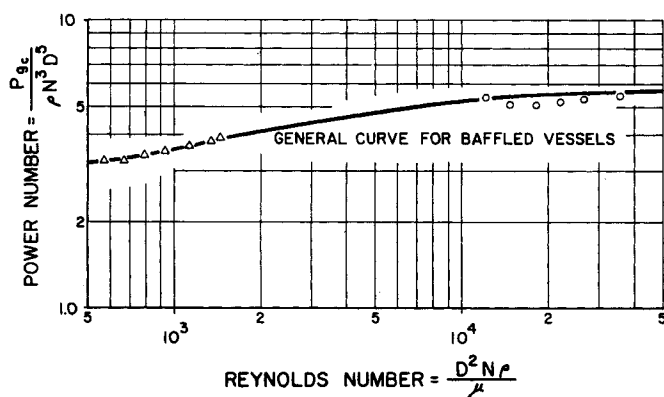


Fig. 6. Correlation of two-phase liquid data points with the general curve for single-phase liquids, agitated batchwise in baffled vessels. 12-in. vessel: no air-liquid interface: same legend as in Fig. 4; air-liquid interface: \bullet S.A.E. 10 motor oil-water, \blacktriangledown blend A-water.



← Fig. 7. Effect of impeller position on Reynolds number and power numbers for two-phase liquids, agitated batchwise in baffled vessels. Water-41%, S.A.E. 10 motor oil-59%; \circ $C = 1.5 D$, impeller at liquid-liquid interface; \triangle $C = 2.0 D$, impeller in oil phase.

was that phase in which the impeller was located when at rest. When the impeller was located at the liquid-liquid interface, the water phase was found to be continuous for oil-water systems and the organic phase for kerosene-water systems.

In Figure 7 the effect is shown of moving the impeller from the light phase to the interface of an oil-water system. The change from a water-in-oil to an oil-in-water dispersion resulted in a lower mixture viscosity, which in turn increased the Reynolds number, the measure of turbulence, by more than twenty-fold.

Continuous-flow Operation

Measurements taken with continuous flow of water and Blend-D-water systems showed negligible deviations from those taken with batch agitation of the same system. Flow was from the bottom to the top of the vessel and the impeller was located above the interface of the two-phase liquid when at rest.

CONCLUSIONS

In summary, the following conclusions are drawn based on agitation in unbaffled vessels with no air-liquid interface and in baffled vessels with four radial baffles each 16.7% of the vessel diameter.

1. Dynamic similarity is obtained in geometrically similar, unbaffled vessels by operating with no air-liquid interface and equal Reynolds numbers.

2. For such unbaffled vessels, a single general curve of N_p plotted against N_{Re} will describe all single-phase and most two-phase liquid data.

with the same mean density and the mean viscosity recommended by Vermeulen [Equation (5)].

7. Impeller height in relation to the liquid-liquid interface of two-phase systems has little effect between one and two impeller diameters from the bottom of an unbaffled vessel.

8. Impeller height in baffled vessels is important in agitation of two-phase liquids. For more effective agitation, the vessel should be operated, if possible, with the impeller in the low-viscosity phase.

9. Continuous flow of liquid through an unbaffled vessel has a small effect on the power characteristics of the impeller. For low flow rates the foregoing conclusions for batch systems can be applied. For baffled vessels the effect of continuous flow is negligible.

NOTATION

- C = impeller distance off tank bottom (to midpoint of blade), ft.
- D = impeller diameter, ft.
- g = acceleration due to gravity, ft./sec.²
- g_c = conversion factor, (ft./sec.²) (lb. mass/lb. force)
- hp. = horsepower
- J = baffle width, ft.
- K = a constant
- N = rotation rate of the impeller, rev./sec.
- N_F = Froude number, DN^2/g
- N_p = power number, $Pg_c/D^5N^3\rho$
- N_{Re} = Reynolds number, $D^2N\rho/\mu$
- P = power, ft.-lb./sec.
- T = vessel diameter, ft.
- x = volume fraction of phase x
- x_c = volume fraction of continuous phase
- x_d = volume fraction of dispersed phase
- y = volume fraction of phase y
- Z = liquid depth, ft.
- μ = liquid viscosity, lb. mass/(ft. (sec.))
- μ_a = weighted geometric mean viscosity [Equation (4)]
- μ_L = mean viscosity derived in this study [Equations (8) and (9)]
- μ_m = mean viscosity of Vermeulen [Equation (5)]
- ρ = liquid density, lb. mass/cu. ft.
- ρ_a = average density [Equation (6)]

Subscripts

- c, d = continuous and dispersed phases
- o, w = organic and water layers
- x, y = phases x and y

Exponents

- m = exponent of the Reynolds number
- n = exponent of the Froude number
- x = volume fraction of phase x
- y = volume fraction of phase y

LITERATURE CITED

1. Flynn, A. W., and R. E. Treybal, *A. I. Ch. E. Journal*, **1**, No. 3, 324 (1955).
- 1a. Laity, D. S., Ph.D. thesis, New York Univ. (1956).
2. Mack, D. E., and A. E. Kroll, *Chem. Eng. Progr.*, **44**, 189 (1948).
3. Miller, S. A., and C. A. Mann, *Trans. Am. Inst. Chem. Engrs.*, **40**, 709 (1944).
4. Overcashier, R. H., H. A. Kingsley, Jr., and R. B. Olney, *A. I. Ch. E. Journal*, **2**, 529 (1956).
5. Rushton, J. H., *Chem. Eng. Progr.*, **47**, 485 (1951).
6. Rushton, J. H., E. W. Costich, and H. J. Everett, *Chem. Eng. Progr.*, **46**, 395, 467 (1950).
7. Vermeulen, Theodore, G. M. Williams, and G. E. Langlois, *Chem. Eng. Progr.*, **51**, No. 2., 85F (1955).

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Froth-frothate Concentration Relations in Foam Fractionation

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Concentration relations between foams and their residual liquids (frothates) have been examined.

Clean air streams of 88, 91, and 96% water saturation were bubbled into aqueous solutions of isobutyl alcohol. The concentration of the alcohol in the collected and collapsed foam was plotted against its concentration in the bulk liquid.

Varying the saturation of the air stream resulted in both positive and negative enrichment of the foam with the surface-active alcohol. This suggests that where froth-frothate-concentration relations are unfavorable, a change in operating conditions may advantageously affect these relations and allow the mixture to be foam separated.

Foam formation is often undesirable—for example, in stills, boilers, or fermenters. As a result, industrial study of foams has largely been directed toward (a) preventing their formation and (b) destroying them once formed. A foam, however, possesses qualities which can be usefully employed in certain instances. One of these is the fractionation of liquid mixtures.

Foaming may be used to separate the components of a homogeneous liquid mixture if one, or more, of them is surface active. When such a liquid is formed, these components may be preferentially adsorbed in the foam layer (froth) and their concentration there will be greater than in either the residual liquid (frothate) or the original solution.

Foaming therefore offers a means of separating positively adsorbed substances from solutions the other components of which are adsorbed to a lesser degree. As such it deserves investigation for it may provide a convenient means of fractionating liquid mixtures, particularly those containing complex, heat-sensitive, and chemically unstable materials. In such cases the more conventional methods, like distillation and extraction, are frequently unsatisfactory.

A few examples of foam separation have been reported. These include the separation of ricinic acids from fatty acids (2), fractionation of fatty acids (1), and the purification of enzymes (5). Ore flotation (7), though apparently quite similar, involves a somewhat different

principle in that the materials being separated are not present in a homogeneous single phase.

PRINCIPLES

Gibbs (3) formulated from thermodynamic considerations an adsorption equation predicting the surface excess of a solute which is in equilibrium with its bulk concentration. In modified form this relationship is

$$\Gamma = -\frac{a}{RT} \left(\frac{d\gamma}{da} \right) \quad (1)$$

where

Γ = excess solute per unit surface

a = solute activity

γ = surface tension of the solution

R = universal gas constant

T = absolute temperature

For sufficiently dilute solutions the mass concentration, x , may be substituted for activity and the equation becomes

$$\Gamma = -\frac{x}{RT} \left(\frac{d\gamma}{dx} \right) \quad (2)$$

Equation (1) says that if the solute is surface active—that is, if it lowers the surface tension of the solvent when added—it will concentrate in the surface layer. Conversely, a surface-inactive solute is negatively adsorbed (or “desorbed”) and its concentration in the interior of the solution is therefore greater than at the surface. In any case, then,

where the addition of solute to a solvent alters the surface tension of that solvent the surface layer of molecular depth will differ in concentration from the solution bulk.

In the work reported here, concentration relationships existing between foams and the bulk liquids from which they were formed have been measured by the rising-bubble technique. Solutions of isobutyl alcohol and water were studied and the results expressed in plots of alcohol concentration in the collected and collapsed foam (froth) vs. concentration in the residual liquid (frothate).

It is important to note that the rising-bubble method does not give equilibrium data. The values obtained are dependent upon the manner in which the bubbling apparatus is operated. Furthermore, in the case reported here the concentrations of froth and frothate were constantly changing, owing to the preferential evaporation of water by the entering air stream. Thus the data presented are for a particular arbitrary operating time (30 min.); longer or shorter periods would have given numerically different results. This point will be considered more completely in the discussion of the experimental data.

With this essential limitation of the rising-bubble technique recognized, it is still the simplest and most direct approach by far.

EXPERIMENTAL DETAILS

Filtered air was bubbled into isobutyl alcohol-water solutions in the apparatus shown in Figure 1. The resulting foam was collected, collapsed, and continuously returned to the solution to be refoamed. This method provides recirculation, making it possible to keep the bubble-retention time constant. If the foam were removed, retention time would continually decrease during the run period. Retention time is defined here as the total fluid volume